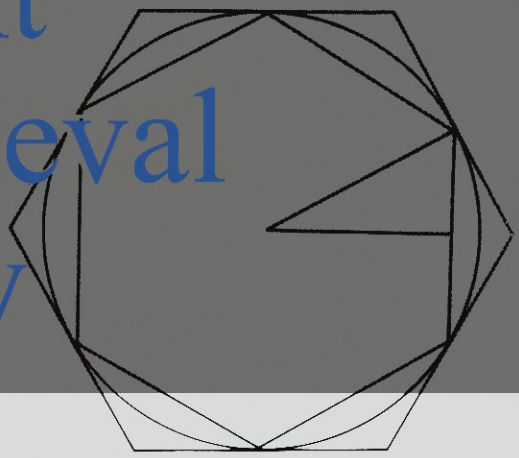
Wilbur Knorr

Textual Studies in Ancient and Medieval Geometry



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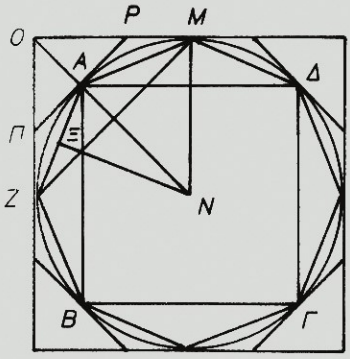

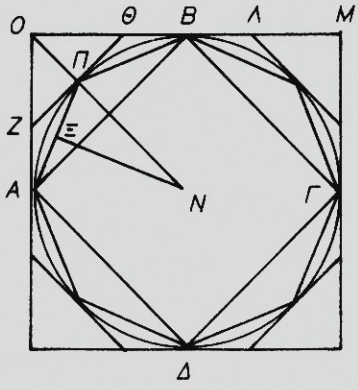

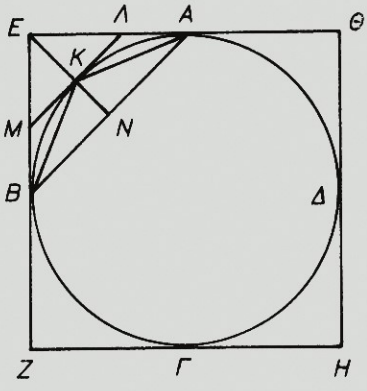

Textual Studies in Ancient and Medieval Geometry

The cube duplication of Naṣîr al-Dîn al-Ṭûsî appears as a marginal comment to the text of Book V, prop. 52 of Apollonius' *Conics* in the Arabic manuscript Bodl. Marsh 667 (f. 106r). For discussion, see part II, chapter A. (Reprinted with permission of the Bodleian Library, Oxford.)

Wilbur Richard Knorr

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To Lawrence V. and Hannah Berman

בְּכָל־עֵת אֹהֵב הָרֵעַ

for their nurturing skepticism, their wit,
and their love of learning

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Although the present work embraces certain textual investigations complementary to my previous volume, *The Ancient Tradition of Geometric Problems* (Birkhäuser 1986), some of its contents have developed from studies begun even before I conceived of that other volume. Specifically, I obtained and studied the Arabic documents on cube duplication and angle trisection, examined in Part II, during a term of research at the Institute for Advanced Study, supported by a grant from the American Council of Learned Societies, in 1978-79, and continued this work under a grant from the National Science Foundation in 1979-80. On the other hand, the materials in Part III, particularly the chapters examining the medieval Archimedean tradition in Arabic, Hebrew and Latin, are the product mostly of recent study. This has been supported in part by grants from the Pew Foundation, administered by Stanford University, in 1984-85 and 1986-87, and by the National Science Foundation in 1987-88.

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To say that the late Lawrence Berman, my colleague at Stanford, and his wife Hannah Berman, who has served as my academic assistant, have provided invaluable guidance in the study of Arabic and Hebrew documents only begins to

describe my debt to them. In dedicating this volume to them, I hope to express in however small a way my thanks for their intellectual, personal and spiritual support.

To the directorates and photographic services of the following library collections, I acknowledge their kind permission to include the facsimiles of manuscript pages that appear in the indicated chapters:

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Columbia University Library (David Eugene Smith Collection of Rare Books and Manuscripts), New York: for reproductions from Arabic ms. Or. 45/4 (see Part III, Appendix to chap. 7)

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Biblioteca nazionale, Florence (Part III, chap. 9)

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University of Wisconsin Press: for reproduction of Marshall Clagett's edition of the medieval Latin versions of Archimedes' *Dimension of the Circle* (see Part III, Appendix to chaps. 3 and 4) from M. Clagett, *Archimedes in the Middle Ages I*, © 1964 by the University of Wisconsin

Deutsche Bibelstiftung, Stuttgart: for reproduction of the half-line of *Proverbs* 17:17 from the *Biblia Hebraica Stuttgartensia*, ed. K. Elliger, W. Rudolph et al., 1967/77 (see the dedication page)

To the following presses I acknowledge their permission for me to present here revised versions of my own recent articles published by them:

Springer Verlag: for a revised version of "Archimedes' *Dimension of the Circle*: A View of the Genesis of the Extant Text" (see Part III, chaps. 1 and 2), *Archive for History of Exact Sciences 35*, 1986

Cambridge University Press: for a revised version of "The Medieval Tradition of Archimedes' *Sphere and Cylinder*" (see Part III, chap. 8), in *Mathematics and its Applications: Essays in Honor of Marshall Clagett*, ed. E. Grant and J.E. Murdoch, 1987

And finally, to the publishers and editors of Birkhäuser Boston Inc., in particular, Lauren Cowles, Elise Oranges and Meike Seeker, I offer thanks for their diligent and efficient efforts.

Wilbur R. Knorr

A Note on Symbols and Transliteration

For convenience of reference, the principal documents discussed in the present work are identified by key letters set in bold face.

In Part I, chap. 1: relative to the texts of the Heronian method of cube duplication from Hero's *Mechanica* (**HM**) and *Belopoeica* (**HB**), the transcriptions by Pappus (**HP**) and by Eutocius (**HE**), and the related methods attributed to Apollonius by John Philoponus (**AJ**) and by Eutocius (**AE**), for full citations, see p. 24, n. 1. These symbols are employed also in chaps. 2, 3 and 5.

In Part I, chap. 3: relative to the texts of the Philonian method of cube duplication from Philo's *Belopoeica* (**PB**), a lost alternative treatment by Philo (**PB***), the transcription by Eutocius (**PE**), and the related method presented by John Philoponus (**PJ**), who attributes a variant (**PK**) of the same to Apollonius, for full citations, see p. 56, n. 1. These symbols appear also in chap. 5. Their relations are schematized on p. 51.

In Part I, chap. 4: the texts of the method of Eratosthenes as transmitted by Pappus and by Eutocius are designated **EP** and **EE**, respectively, on pp. 67-68; for the translations and source references, see chap. 6, pp. 146-150.

In Part I, chap. 5: for the method of Diocles (**D**) and its adaptation by Eutocius (**DE**), see pp. 81-82 and 120, n. 25; for the methods of Pappus (**P**) and Sporus (**S**), see pp. 87-88 and 122, n. 64; for the method of Archytas as given by Eutocius (**E**) and by the Banû Mûsâ (**BM**), see pp. 100-110, 126n123, and 127n132.

In Part I, chap. 7: relative to the texts on compound ratio by Eutocius in his Archimedes commentary (**A**) and his Apollonius commentary (**C**), see p. 170, n. 13 for source references, and for translations and texts, see the Appendix; for the version in the anonymous *Introduction to the Syntaxis* (**B**), in the Venice ms. (**BV**) and the Paris ms. (**BP**), see pp. 170n13, 185 and 190 for citations, and for translation and text, see the Appendix; for the treatment by Domninus (**D**), the source citation, translation and text appear in the Appendix, pp. 201-207.

A new scheme of symbols is introduced for the Arabic documents and related Greek sources discussed in Part II (for full citations of the sources, see p. 249):

A denotes the cube duplication of Abû Bakr al-Harawî, ancillary documents are **A2** (in Greek, same as **PK** of Part I), **A3** (Abû Jaᶜfar), **A4** (al-Ṭûsî), **A5** (*Conics* II 8, Arabic and Greek) and **A6** (*Conics* IV 30 in Arabic and IV 35 in Greek).

B denotes the angle trisection of Aḥmad ibn Mûsâ, **B*** an alternative ms. of the same, and **B2** one of the angle trisections given by Pappus (in Greek).

C denotes the angle trisection of Thâbit ibn Qurra, while ancillary documents, both in Arabic and Greek, are **C2** (*Conics* II 4), **C3** (*Conics* II 12), and **C4** (the hyperbola construction of Eutocius).

D denotes the angle trisection by al-Sijzî, **E** the cube duplication and angle trisection by al-Qûhî, **F** the rendition by Abû Jaᶜfar of Nicomedes' cube duplication, where Nicomedes' version itself is denoted **F2** in its Arabic version by Thâbit, and **F3** in the Greek.

In Part III a third scheme is followed, where, in general, once a symbol is introduced, it recurs in most or all of the subsequent chapters. For Archimedes' *Dimension of the Circle* and *Sphere and Cylinder*, the symbols in italics, *DC* and *SC*, respectively, are used when the discussion bears on the general tradition of the work, without reference to a specific form or version of it.

In chaps. 1 and 2, for the Greek line of Archimedes' *Dimension of the Circle*, the following symbols are introduced: **DC** represents the form extant in the Archimedean corpus, **P** Pappus' version and **T** Theon's version (see p. 396, n. 7 for sources), and **S** is Pappus' theorem on sectors (see p. 384); lost prototypes of the circle theorem are designated with asterisks: **P*** is an alternative version due to Pappus (see p. 381), **A*** its older Archimedean prototype (see p. 386), and **DC*** a direct antecedent of the extant Greek, reconstructed from the medieval versions (see p. 422). For the relation of these text forms, see the table on p. 405.

The medieval versions of *Dimension of the Circle*, discussed in chaps. 3 and 4, include versions in Arabic (**AF**) and Hebrew (**H**), and Latin versions by Plato of Tivoli (**LP**) and Gerard of Cremona (**LG**); see pp. 421, 436. Their interrelations are schematized on p. 431. Comparisons among the forms of *Sphere and Cylinder* extant in Greek (**SC**), Arabic (**ASC**) and Hebrew (**HSC**) are made on pp. 442-450.

In chap. 6, the extant form of Eutocius' commentary on Archimedes is denoted **E**, and its original form (now lost) **E***; see pp. 526-527.

In chap. 7 are discussed the Arabic adaptations of Archimedes by the Banû Mûsâ (**BM**), Abu 'l-Rashîd (**AR**) and al-Ṭûsî (**AT**); see p. 535.

In chap. 8 the medieval tradition of *Sphere and Cylinder* examines the accounts by the Banû Mûsâ in its Latin version, *Verba filiorum* (**VF**), and by John of Tynemouth in the Latin *De curvis superficiebus* (**CS**); see pp. 595-596. The proposed lost Greek prototype of the latter is denoted **CS*** (see p. 608). Comparisons are made with parts of the anonymous tract on isoperimetric figures (**AI**), from the *Introduction to the Syntaxis* (p. 607).

In chap. 9 the medieval tradition of *Dimension of the Circle* is surveyed. Based on Gerard's translation (**LG**, as in chap. 3), it embraces at least ten paraphrase editions: versions in Florence (**F**), Cambridge (**C**) and Naples (**N**) manuscripts, the Gordanus version (**Go**), the Munich version (**M**), the pseudo-Bradwardine, or Vatican, version (**V**) and its abbreviated form (**AV**), the *Questio* of Albert of Saxony (**QA**), and versions in Corpus Christi (**Co**) and Glasgow (**Gg**) mss.; see pp. 617-618. Comparisons are also made with **CS** (see chap. 8) and a paraphrase version of it ($\mathbf{CS_2}$); see p. 628. The relations among these versions are schematized on p. 668.

In chap. 10 the anonymous isoperimetric tract (**AI**, as in chap. 8) is compared with forms extant in Pappus (**PI**) and Theon (**TI**), and these are related to their presumptive sources: a lost version by Pappus (**PI***), the lost tract by Zenodorus (**Z***), and a lost version (**AI***) exploited directly by the anonymous editor; see pp. 690-691 and 710.

A comprehensive diagram for the ancient and medieval Archimedean tradition appears on p. 806.

For Greek words given in transliteration, I employ circumflex for the long vowels *êta* and *ômega*.

For Arabic words in Part II, I follow the transliteration system of Wehr and Cowan, *Arabic-English Dictionary*, 3rd ed., save that for certain letters with inconvenient diacritical markings (specifically, those with underbars: $\underline{k}$, $\underline{t}$, $\underline{d}$, and with háček: $\check{s}$) I usually prefer the alternative forms with *h* (that is, *kh*, *th*, *dh*, *sh*). Long vowels are denoted by macrons ($\bar{a}$, $\bar{u}$, $\bar{\imath}$). But in Parts I and III, I mark long vowels with circumflex ($\hat{a}$, $\hat{u}$, $\hat{\imath}$), to be consistent with the Greek transliterations there.

Hebrew words appear in Part III, chap. 3, Appendix III. I employ the standard transliteration for the consonants, roughly equivalent to the cognate Arabic letters (cf. *Gesenius' Hebrew Grammar*, ed. E. Kautzsch, in the 2nd English edition by A.E. Cowley, Oxford, 1910, p. 26); long vowels are written with circumflex when *plene*, but with macron when *defective* (*ibid.*, pp. 40-41). For Hebrew names, however, I omit diacriticals.

In the lettering of Arabic and Hebrew diagrams I adopt a modified system to conform with the implied Greek base: thus, the usual sequence is *A, B, G* (for *j*), *D, E* (for *h*), *Z, H* (for $\underline{h}$), *T* (or θ, for $\underline{t}$), *I* (for *y*), *K, L, M, N, X* (for *sh*), *O* (for ᶜ). When they appear, *W* (or *U*), *F, Q, R, S,* $\underline{S}$, $\underline{D}$, $\underline{Z}$ stand for the regular Arabic letters (cf. pp. 80, 437, 555, 556, 577). In the unusually elaborate figures employed in **AR** (pp. 555-556), I have had to improvise somewhat, writing *t* (for the regular *t*, since *T* here stands for $\underline{t}$), *k* (for *kh*, since *K* here stands for the regular *k*), *k'* (for a second *kh*) and *C* (for the regular *th*); in these cases, the reader should check against the facsimiles on pp. 566 and 568.

Table of Contents

Acknowledgments vii

A Note on Symbols and Transliteration xi

Introduction: Philologist, Heal Thy Text 1

Part I Ancient Texts on Geometric Problems

1 The Hero-Apollonius Method of Cube Duplication 11

2 The Hero-Apollonius Lemma in Nicomedes and Euclid 29

3 The Philonian Method of Cube Duplication 41

Appendix: Philoponus' Account of Cube Duplication 59

4 Pappus' Texts on Cube Duplication 63

5 Eutocius' Anthology of Cube Duplications 77

The Platonic Construction (78) The Methods of Hero, Philo, and Apollonius (81) The Method of Diocles (81) The Methods of Pappus and Sporus (87) The Method of Menaechmus (94) The Method of Archytas (100) The Methods of Eratosthenes and Nicomedes (111) Summary (111)

6 Eutocius' Text of Eratosthenes: A Thesis of U. von Wilamowitz 131

Appendix: Two Accounts of Eratosthenes' Method 146

7 On Eutocius: A Thesis of J. Mogenet 155

Appendix: Four Texts on Compound Ratio 177

8 Angle Trisections in Pappus and Arabic Parallels 213

9 The Ancient Commentators and Their Methods: Pappus and Eutocius 225

Part II Arabic Geometric Texts and Their Ancient Sources

A The Cube Duplication by Abû Bakr al-Harawî 251

B The Angle Trisection by Aḥmad ibn Mûsâ 267

C The Angle Trisection by Thâbit ibn Qurra 277

D The Angle Trisection by al-Sijzî 293

E The Cube Duplication and Angle Trisection by Abû Sahl al-Qûhî 301

F The Cube Duplication by Abû Jaᶜfar in the Manner of Nicomedes 311

Appendix: Texts in Transcription (320) and Facsimile (352)

Part III The Textual Tradition of Archimedes' *Dimension of the Circle*

1 Versions in the Ancient Commentators 375

Appendix I. Archimedes' Circle Theorem in Pappus and Theon: Translations and Facsimiles (387) II. Pappus' Text of the Sector Theorem: Translation and Facsimile (394)

2 Origin of the Extant Text of the *Dimension of the Circle* 401

Appendix I. The Extant Greek Text of *Dimension of the Circle*: Translation and Facsimile (411) II. Theon's Lemma to the Circle Theorem: Translation and Facsimile (414)

3 The Medieval Tradition of *Dimension of the Circle*, Prop. 1 421

Appendix I. Translation from the Arabic Text (436) II. Variant Readings (438) III. On the Arabic and Hebrew Translators (441)

Facsimiles of the Arabic, Hebrew and Latin Texts 455

4 Versions of *Dimension of the Circle*, Props. 2 and 3 477

Appendix I. The Ancient and Medieval Texts of Props. 2 and 3 (484) II. Variants in the Medieval Versions (489)

5 Lost Propositions of the Archimedean Prototype 495

6 Eutocius' Text of *Dimension of the Circle* 513

Appendix: Extracts from Eutocius and Theon 533

7 Arabic Elaborations of the *Dimension of the Circle* 535

Banû Mûsâ (535) Abu 'l-Rashîd (543) al-Ṭûsî (546)

Appendix (551) I. The Version of Abu 'l-Rashîd: Translation and Facsimile (552) II. The Version of al-Ṭûsî: Translation and Facsimile (577)

8 The Latin Tradition: *De curvis superficiebus* 595

9 The Latin Tradition: *De quadratura circuli* 617

The Florence Version (618) The Cambridge Version (625) The Naples Version (631) The Gordanus Version (638) The Munich Version (640) The Vatican, or ps.-Bradwardine, Version (644) The Abbreviated Version (648) The Version by Albert of Saxony (649) The Corpus Christi Version (655) The Glasgow Version (663) Synthesis (668)

10 The Anonymous Tract *On Isoperimetric Figures* 689

Appendix: Propositions of the Anonymous Tract 738

11 On Hypatia of Alexandria 753

Appendix I. Four Accounts of Division from Theon's Commentary on Ptolemy (780) II. Pappus' Method of Long Division (787) Facsimiles from Theon (802)

12 The History of a Text: Tradition, Time and Opportunity 805

Bibliography 817

Index 833

Introduction: Philologist, Heal Thy Text

> Healing is a matter of time,
> but it is sometimes
> also a matter of opportunity.
> (Hippocratic *Precepts*, chap. 1)[1]

To the classical scholar, unlike the ancient physician, time is the enemy, for time—far too much of it—has afflicted the documentary record of antiquity with all manner of ills. Indeed, the metaphor of disease pervades the language of textual criticism: one speaks of the "corruption" that texts suffer in the course of their transmission, even as the purity of one textual form is "contaminated" or "infected" by defects from another. In the words of a distinguished guide to classical scholarship,

> The business of textual criticism is in a sense to reverse this process, to follow back the threads of transmission and try to restore the texts as closely as possible to the form which they originally had.[2]

At times this goal is unattainable, and the text must be diagnosed as "incurable."[3] But to the extent it is possible, the philologist, like a physician, takes on the role of "restorer" of texts.

This description captures, I think, an essential aspect of the philologist's task, as commonly conceived within the field. In effect, the aim of philology is defined in terms of its *product*, the establishment of textual prototypes. This aim was already evident among the Renaissance humanists, but only in the 19th century were the methods of textual analysis refined to attain it, by systematizing procedures and minimizing the role of subjective judgments. Basically, through the critical examination of the extant manuscripts of a given work, one determines those that appear the most trustworthy (not necessarily the earliest), hence reflective of the oldest accessible form of the text. Through their collation one seeks to construct a text that is the closest possible approximation to the original form, but, where the evidence is questionable, to identify among the variants those most likely to be candidates for the original reading. At the same time, one seeks to identify passages likely to be later accretions—interpolations and other editorial changes—that in the course of a text's transmission have come to spoil its pristine form.

For textual studies relating to the ancient mathematical corpus the efforts by the Danish philologist, J.L. Heiberg (1854–1928), are especially significant. Beginning with his doctoral dissertation, *Quaestiones Archimedeae* (Copenhagen, 1879), Heiberg produced an astonishing series of editions and critical studies that remain the foundation of scholarship on Greek mathematical science.[4] For comprehensiveness and accuracy, his editions are exemplary. In his textual studies, as also in the prolegomena to his editions, he carefully described the extant evidence, organized the manuscripts into stemmata, and drew out the implications for the state of the text.[5]

With regard to his Archimedean work, Heiberg sometimes betrayed signs of the philologist's occupational disease – the tendency to rewrite a text deemed on subjective grounds to be unworthy.[6] But he did so less often than his prominent contemporaries, and not as to detract appreciably from the value of his editions.[7] In examining textual questions bearing on the Archimedean corpus, he attempted to exploit as much as possible evidence from the ancient commentators, and in some instances from the medieval translations. It is here that opportunities abound for new work, extending, and in some instances superseding, Heiberg's findings. For at his time the availability of the medieval materials was limited. In recent years Marshall Clagett has completed a mammoth critical edition of the medieval Latin tradition of Archimedes,[8] while the bibliographical instruments for the Arabic tradition are in good order thanks to the work of Fuat Sezgin.[9] In the light of the medieval resources one finds in many quite surprising ways that positions assumed by Heiberg, particularly in connection with *Sphere and Cylinder* and *Dimension of the Circle*, must be radically modified.

But *emendatio textuum* need not be the sole objective of text studies. Recalling the ancient medical paradigm, although restoring health is one's primary aim, one must nevertheless acknowledge the limitations of one's expertise. Even where the range of intervention is restricted – indeed, even where the prognosis is hopeless – one does not cease observing and attempting to make sense of what is observed. For whatever reasons, understanding the course of a disease, even an incurable one, merits one's effort. In effect, pure intellectual curiosity has its own rewards. The project of understanding the phenomenon of disease – the nature and causes of its progress, whether in men or in texts – that is, of tracing out a historical *process*, takes its place beside that of mastering this process and reversing the product of its course. The knowledge thereby obtained can be hoped, in subtle and unanticipatable ways, to contribute eventually to the restorer's art.

I propose in the chapters that follow to trace out the transmission and development of a specific set of ancient mathematical works, in effect, to compile a redaction history of them, to the extent that the surviving evidence makes possible. I will consider first an assortment of texts bearing on the problems of cube duplication and angle trisection: in Part I, the ancient evidence dominates, most notably in the expositions of these problems by Pappus and Eutocius; in Part II, selections from the medieval Arabic evidence will be considered, with the aim of articulating their relation to ancient prototypes, extant or lost. In Part III I will focus on the tradition of a single work, Archimedes' *Dimension of the*

Circle: I will first survey its treatment by the ancient Greek commentators, then its translation into Arabic, Hebrew and Latin and its subsequent study in the Middle Ages.

One can expect that detailed overviews of the extant tradition of works like these will permit a better informed use of the extant materials more generally:

First, one will possess a body of specimens of the alterations such texts undergo in the course of their transmission. The documents here all relate to a particular genre, the formal exposition of geometric propositions (theorems or problems). Evidence for any given item can take a variety of forms. The majority of citations of such results, for instance, as by ancient handbook writers or commentators, are casual statements (sometimes in paraphrase) of the enunciation, without the proof. When a version of the full construction or proof is given, the source version may be reproduced precisely in full, or it may be abridged, amplified or otherwise modified. One might suppose that new editions of a work will adhere closely to the source, but even here editorial changes of a similar kind are common. Additional changes enter when a work is translated. From there, the text typically undergoes further change by editors and commentators within the daughter tradition. One can thus gain a sense of the relative frequency of minor changes such as substitutions of terminology, additions or deletions of single steps, attachment of explanatory remarks, changes of order, and the like, as well as identify instances where editorial intervention has been more substantial.

Second, the characteristic manner of the principal editors can be inferred, through comparison against the assembled evidence for each geometric effort. Thereafter, if one encounters some version of a proof, say in Pappus or Theon or Eutocius, one will have a sense of how close this version is likely to be to the editor's source. Since some of these figures (Theon and Eutocius, in particular) affected the tradition of the principal treatises (as editors of Euclid and Apollonius, respectively), insight into their methods is indispensable for proper use of these extant editions.

Third, reverting to the therapeutic aim, when we have separated out the editorial changes that have entered into the tradition of a given work, we can infer the manner of the underlying prototype. From the survey of versions of *Dimension of the Circle* in Part III, for instance, we will discover that, contrary to the supposition of Heiberg and other critics, the received text is only remotely related to Archimedes' original. Hallmarks of individual editors emerge (for instance, a line like "it has been proved that this of necessity can be done" is certain to be a write-in by Pappus), so that when these are encountered, we can confidently distinguish them from the editors' sources; when such phrases occur frequently in a text, we can develop a case for attributing it to a particular editor. Remarkably, the process that would have been thought to introduce the greatest changes, namely, that of translation, tends actually to introduce the least. For, as a rule, the translators—whether rendering from Greek into Latin or Arabic, or from Arabic into Latin or Hebrew—strive for verbatim agreement with their sources. Their policy of literal rendering results in documents of considerable

value for the study of ancient texts. Although I will not attempt the reconstruction of any prototype, the way to do that will become evident in several cases through the evidence compiled here.

To discuss the tradition of any document, one must first establish the source relations linking each version to the others within that tradition. To this end, I will apply a method of literary analysis similar to that familiar among philologists in their efforts to set out the genealogical relations linking manuscripts. Indeed, I will often attempt to produce branching diagrams resembling the philologist's *stemmata*. But a basic distinction must be noted. The philologist is usually engaged in collating variant readings among copies of the same work. Since scribal error introduces a certain amount of discrepancy at each stage, *differences* become a measure of the *distance* separating given copies. Agreement, on the other hand, will be expected for most of the text, so that only agreement on unusual variants will be considered significant. In our case, however, the document base consists of edited versions and paraphrases. In general, the editorial latitude is quite high—at least, in the specific textual expressions presented, if not in the technical execution of the proofs—so that textual differences are generally of little significance. On the other hand, *similarities* become a measure of *affiliation*. For if edited versions happen to agree on certain details—e.g., the use of distinctive expressions, or the structuring of proofs where the order of steps is arbitrary—then one derives the basis for arguing some form of textual dependence. Upon further scrutiny, this may support claims that one version is the immediate source for another, or that two versions are in parallel dependence on a third (possibly not extant).[10]

The same literary method is useful for investigating questions of authorship and authenticity. By comparing a work of unknown or uncertain attribution against recognized works of a given writer, one can measure the likelihood of common authorship. We will scrutinize several cases of this sort: a thesis of U. von Wilamowitz pronouncing spurious the letter of Eratosthenes transmitted by Eutocius (Part I, chap. 6); a thesis of J. Mogenet assigning to Eutocius authorship of the *Introduction to the Syntaxis* (chap. 7); a proposal will be set out on the pattern of source relationships or common editorial hand among the works in the medieval Latin tradition of Archimedes (Part III, chap. 9); and another proposal will be made on the lost works of Hypatia (chap. 11).

Admittedly, a subjective element enters in such analyses, for judgments on the significance we should assign to given textual coincidences well may differ from one text critic to another. I recognize that a certain amount of disagreement will be inevitable, and that this must condition the persuasiveness of some of my conclusions. My aim will be to present the documents and my interpretations of them. Although others may wish to entertain alternative hypotheses, one should note that those I offer represent my own estimate of the best possible accounts among the many that seem possible.

In my *Ancient Tradition of Geometric Problems* I have surveyed the technical features of this material and the metamathematical observations made by the ancient commentators, so I will cite that book when such matters arise here. On

technical matters the writers of late antiquity are generally not notable for innovations anyway—they seem to have held the repertory of methods and results to be complete, through the researches of Euclid, Archimedes, Apollonius and their associates in the heyday of geometry, the 3rd century B.C. Even where the later writers volunteer new results, one can readily appreciate their derivative character; examples of this sort will arise in the course of Part I (e.g., the cube duplications by Hero in chap. 1, by Pappus and Sporus in chap. 5, the various angle trisections in chap. 8, and so on).

Among the works by the ancient Greek commentators of particular interest in the present study are the following:

Hero of Alexandria (mid-first century A.D.) composed several widely used handbooks in mechanics and geometry—e.g., the *Mechanics*, the *Belopoeica* (on artillery), the *Metrica*, and others. Hero relies heavily on Archimedean materials, and sometimes preserves evidence of lost works. (Part I, chaps. 1 and 3; Part III, chaps. 1, 4 and 5)

Menelaus of Alexandria (early 2nd century A.D.) is best known for the *Sphaerica*, a treatise on spherical geometry, utilized by Ptolemy as the foundation for the geometric methods of mathematical astronomy (cf. *Syntaxis* I 12–13). He also produced a work, sometimes cited as the *Elements of Geometry* (of which only a few fragments in Arabic survive), a compilation of geometric results that served as a source for later commentators. (Part I, chaps. 3 and 5; Part II, chap. C)

Pappus of Alexandria (early 4th century) is best known to us for the massive *Collection*, of which most of books II through VIII are extant; this preserves a wealth of material on the ancient geometric tradition, much of which would otherwise be unknown to us. But as the leading teacher of mathematics and astronomy at Alexandria, Pappus was most influential in his own time for his commentaries on Ptolemy and Euclid. (Part I, chaps. 1, 2, 4, 6, 8, and 9; Part II, chaps. B and C; Part III, chaps. 1 and 10)

Theon of Alexandria (mid-4th century) was Pappus' successor as professor of mathematics and astronomy at Alexandria, and his own commentary on Ptolemy was heavily endebted to Pappus. As technical editor, Theon produced new versions of Euclid's works (in particular, the *Elements* and *Optics*) that virtually superseded the older recensions. His commentaries appear also to have influenced the tradition of Archimedes. (Part III, chaps. 1–4, 10 and 11)

Hypatia of Alexandria (late 4th century; d. 415), Theon's daughter and successor as professor of mathematical studies, was a prominent teacher of philosophy, being the principal exponent of Neoplatonism in her generation. The titles of her philosophical works, if indeed she wrote any at all, have not been transmitted, but she is reported to have composed commentaries on Apollonius, Diophantus and other standard mathematical works. (Part III, chap. 11)

Proclus (c. 410–485), disciple of Syrianus, was leader of the Neoplatonist school at Athens. His commentary on Euclid's Book I is a valuable resource for philosophical, historical and technical background on elementary geometry. A participant in the pagan religious revival of late antiquity, Proclus continued the tradition of Iamblichus, as in his commentary on Plato's *Timaeus*. (Part I, chap. 7; Part III, chap. 10)

Eutocius of Askalon (early 6th century) studied philosophy with Proclus' disciple Ammonius at Alexandria. He produced commentaries on Archimedes and Apollonius which greatly influenced the subsequent manuscript traditions of these authors. His colleagues and followers, such as Anthemius of Tralles and Isidore of Miletus, were notable in the advancement of mathematical studies at Constantinople later in the 6th century. (Part I, chaps. 1–3, 5–7 and 9; Part III, chaps. 6 and 11)

John Philoponus and *Simplicius* (early 6th century), like Eutocius, studied under Ammonius at Alexandria. Philoponus, a prominent Christian philosopher, attacked the Neoplatonist cosmology of Proclus, but also contributed to Aristotle studies through his commentaries on the logical and physical writings. (Part I, chaps. 1 and 3) Simplicius, a pagan, also compiled major commentaries on Aristotle, most notably on the *Physics* and *De caelo*. (Part III, chap. 10)

The Arabic tradition of mathematical study will be of interest here primarily for the translations of ancient technical works produced at the Baghdad court of caliph al-Ma'mûn and his successors in the 9th century. Notably active among the translators were *Thâbit ibn Qurra* (Part I, chap. 5; Part II, chap. C; Part III, chap. 3), the *Banû Mûsâ* – that is, the "sons of Mûsâ," Muḥammad, Aḥmad and Ḥassan (Part I, chap. 5; Part II, chap. B; Part III, chaps. 7 and 8), and *Qusṭâ ibn Lûqâ* (Part III, chap. 3). Special mathematical studies, illustrating the use of translated materials, will be considered from *Abû Bakr al-Harawî* (9th century; Part II, chap. A) and *al-Khâzin* (10th century; chaps. A and F), as also from *al-Sijzî* and *al-Qûhî* (late 10th century; chaps. D and E). Editions and commentaries on Archimedes' circle measurement will be examined from the Banû Mûsâ, *Abu 'l-Rashîd* (date uncertain), and *al-Ṭûsî* (13th century) in Part III, chap. 7.

From the Hebrew technical tradition, we will consider translations by *Moshe ibn Tibbon* and *Ya'akov b. Makhir* (late 13th century) and *Qalonymus b. Qalonymus* (early 14th century) in Part III, chap. 3. The medieval Latin tradition is represented in the Archimedes translations by *Plato of Tivoli* and *Gerard of Cremona* in the 12th century (Part III, chaps. 3 and 4) and the paraphrase versions by *John of Tynemouth* (13th century) and a group of anonymous editors (chaps. 8 and 9).

It is clear that the writers and works surveyed here satisfy most of the familiar senses of the term "tradition." The ensemble of texts and textual variants bearing on any given item (say, a particular method for the solution of a problem) tend to preserve a basic form, and often details of wording, so that genealogical affinities can be surmised. When the item is a tract, one typically encounters a similar uniformity among the copies, recensions, paraphrases and elaborations, such that the editorial contributions can be set against a recognizable nucleus. Beyond texts, however, traditions can be delimited also in terms of individuals. Teachers transmit to their disciples a body of learning and techniques, a consensus on issues of significance and the methods appropriate for investigating them. Generations of scholars identified with the paragons of their traditions, like Plato and Archimedes. Ironically, the office of personal succession can sometimes give rise to considerable freedom in the treatment of texts: the perceived essence is maintained, even as the verbal package is transformed.

In its positive sense "tradition" entails continuity and stability, but it also carries the pejorative connotation of stagnation. The present context of textual studies lends prominence to the former aspect, as one comes to appreciate how the ancient and medieval commentators preserve witness to the original form of writings despite barriers of time and language. But the modern reader might be struck more by the derivative and non-innovative character of the commentators' activities. Doubtless, this impression is accentuated by my selection of documents, for the kinds of text to be examined – editions and paraphrase versions based on standard sources – do not constitute a genre marked by novelty; they were intended to facilitate the study of the classical writings within the mathematical curriculum of the ancient and medieval schools. One would not anticipate a high level of originality within this medium, just as one does not anticipate it in modern textbooks. But in the modern case original research is indeed undertaken, whereas little evidence of comparable activity is to be found among the writers of late antiquity. Thus, the differences between the ancient and modern enterprises of scholarship are real.

One would hesitate to render the same judgment in the case of the medieval Arabic tradition, where many of the extant writings incorporate findings from original research. But even here, one must be careful not to assume that the absence of a known ancient prototype implies the originality of the medieval effort. The documents surveyed in Part II, for instance, reveal the influence of ancient sources, even where that is not explicitly acknowledged in the texts themselves. Further, the very gaps in the transmission of ancient materials sometimes stimulated efforts at reconstruction or elaboration.[11] In effect, new work of this sort is an extension of the older tradition through its conscious recreation and imitation.

By sorting out the strands of tradition we will aim to document how certain mathematical texts have been studied and edited, and through these cases to understand better the process of textual transmission. At the same time, we will obtain evidence permitting inferences about the existence and form of lost prototypes. In this way, the more familiar aims of the philologist, the restitution of texts, will be advanced. While time is not with us, opportunity sometimes is. I hope thus to display how materials not often exploited in this context – the testimony of ancient and medieval commentators and translators – can be a resource for the study of ancient texts.

Notes to the Introduction

[1]Cf. the edition of the Hippocratic writings of W. H. S. Jones (Loeb Classical Library), I, pp. 312–313.

[2]L. D. Reynolds and N. G. Wilson, *Scholars and Scribes*, p. 186.

[3]Cf. the annotation sometimes attached to defective readings, *locus nondum sanatus*. For an interesting application of the medical metaphor, see the remark by Heiberg in note 7.

[4]For a summary of Heiberg's life and work, see the obituary notice by C. Höeg, *Jahresbericht über die Fortschritte der klassischen Altertumswissenschaft*, 1931, 75 (233), "Nekrologe," pp. 38–77. Höeg appends a list of Heiberg's "most important works," that is, his editions and major studies – among these his editions of Euclid, Archimedes, Apollonius, Ptolemy, minor mathematical writers (such as Theodosius, Hero, Serenus, and Anthemius), philosophical commentators (Simplicius), and medical writers (Hippocrates and Paul of Aegina). Heiberg's many popular pieces (numbering in the hundreds, mostly in Danish) are not included in this list (*ibid.*, p. 72n).

[5]For my purposes Heiberg's *Philologische Studien zu griechischen Mathematikern* (five papers in the *Jahrbücher für classische Philologie*, 1880–84) have been particularly valuable. These include, among other things, seminal inquiries into the work of Eutocius and the textual history of Archimedes' *Sphere and Cylinder.*

[6]In the case of several works, like *Spirals*, *Plane Equilibria* and *Quadrature of the Parabola*, Heiberg "restored" Doric dialect readings against the evidence of the manuscripts. (Only the total loss of the Doric forms prevented him from doing the same for *Sphere and Cylinder* and *Dimension of the Circle.*) In view of the large number of changed readings, Heiberg noted them not in his regular *apparatus*, but in a separate index at the beginning of the volume (*Arch. Op.*, II, pp. xi–xviii); the index is cumbersome to use and complicates the effort to distinguish authentic readings from secondary additions by copyists and glossators.

[7]Heavy-handedness is in keeping with the 19th century philological style, as one can see from Hultsch's edition of Pappus and Tannery's edition of Diophantus. Indeed, Heiberg himself criticized Hultsch for "employing the knife where the harm could have been healed by the milder cure of emendation" (*Phil. Studien* V, p. 566). Here, "cure" (*Cur*) may have the connotation of treatment by the baths.

[8]*Archimedes in the Middle Ages* has been issued in five enormous volumes, 1964–1984. The present discussion will refer most frequently to the texts on the Arabo-Latin tradition, in Clagett's vol. I.

[9]Of the many volumes of Sezgin's *Geschichte des arabischen Schrifttums*, the one on mathematics (vol. V, 1974) will be of exclusive interest for me here. The predecessor work by H. Suter, *Mathematiker und Astronomen der Araber* (1900), while superseded to a considerable extent, still holds valuable information, particularly on the later period (after 1050) not covered by Sezgin.

[10]I have already explored the possibilities of comparative analyses of this kind in my "Ancient Versions of Two Trigonometric Lemmas" and "The Medieval Tradition of a Greek Mathematical Lemma." These studies also illustrate the domain of validity of a general methodological criterion, the hypothesis of a single source, that is applied in the present work (see in particular, Part I, chaps. 1 and 3, and Part III, chap. 9).

[11]A notable example is ibn al-Haytham's *Completion of the Conics* (cf. the recent edition by J. P. Hogendijk, 1985), an attempt to reconstruct the lost Book VIII of Apollonius' *Conics*. A more modest instance is evident in al-Sijzī's discussion of angle trisection (Part II, chap. D).